GENERALIZED THEORY OF UNCERTAINTY (GTU)—A TRIBUTE TO PROFESSOR G. KLIR

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BACKDROP

INFORMATION AND UNCERTAINTY

- Uncertainty is an attribute of information.

GTI

Klir

generalized theory of information

measure of information

GTU

Zadeh

generalized theory of uncertainty

meaning of information

Minsky/Bar-Hillel

PREAMBLE

- The real world is a world in which uncertainty and imprecision are ubiquitous. There is a deep-seated tradition in science to draw on probability theory to deal with uncertainty and imprecision. Clearly, the usefulness of probability theory is beyond question. But is probability theory sufficient for dealing with uncertainty? A widely held view within the probability community is that it is. To quote Professor Dennis Lindley, an eminent Bayesian:

CONTINUATION

- The only satisfactory description of uncertainty is probability. By this I mean that every uncertainty statement must be in the form of a probability; that several uncertainties must be combined using the rules of probability; and that the calculus of probabilities is adequate to handle all situations involving uncertainty.

CONTINUATION

- *Probability is the only sensible description of uncertainty and is adequate for all problems involving uncertainty. All other methods are inadequate…anything that can be done with fuzzy logic, belief functions, upper and lower probabilities, or any other alternative to probability can better be done with probability.*

CONTINUATION

- ***What is widely unrecognized is that standard probability theory, call it PT, has a serious limitation. More specifically, PT is based on bivalent logic—a logic which is intolerant of imprecision and does not admit shades of truth and possibility. As a consequence, the conceptual framework of PT is not the right framework for dealing with imprecision and, more particularly, with imprecision of information which is described in natural language.***

CONTINUATION

- ***Basically, a natural language is a system for describing perceptions. Perceptions are intrinsically imprecise, reflecting the bounded ability of sensory organs, and ultimately the brain, to resolve detail and store information.***

CONTINUATION

- *Imprecision of perceptions is passed on to natural languages. This is the reason why natural languages are intrinsically imprecise.*

- *Imprecision of natural languages severely limits the ability of PT to compute with information which is described in natural language. In short, PT does not have NL-capability.*

CONTINUATION

- ***This limitation is a serious shortcoming of PT since much of human knowledge is described in natural language. And, as we move further into the age of machine intelligence and mechanized decision making, the capability to compute with information which is described in natural language acquires greater and greater importance.***

CONTINUATION

- ***Viewed in this perspective, the incapability of PT to compute with information described in natural language is a major problem.***

CONTINUATION

- *To endow probability theory with NL-capability, it is necessary to shift the foundation of probability theory from bivalent logic to fuzzy logic. Thus, the generalized theory of uncertainty (GTU) may be viewed as a step in the direction of replacing bivalent logic with fuzzy logic as a foundation for probability theory.*
- *The centerpiece of GTU is the concept of a generalized constraint.*

FUZZY LOGIC—KEY POINTS

- *"Fuzzy logic" is not fuzzy logic*
- *Fuzzy logic is a **precise** logic of imprecision*

The principal distinguishing features of fuzzy logic are:

a) *In fuzzy logic everything is, or is allowed to be graduated, that is, be a matter of degree or, equivalently fuzzy*

FUZZY LOGIC—KEY POINTS

b) *In fuzzy logic everything is allowed to be granulated, with a granule being a clump of points drawn together by indistinguishability, similarity or proximity*

- *Graduation and granulation play essential roles in human cognition*
- *Fuzzy logic is inspired by the remarkable human capability to converse, reason and make rational decisions in an environment of imprecision, uncertainty, incompleteness of information and partiality of truth.*

IMPRECISE PROBABILITIES

- Real world probabilities are, for the most part, imprecise.
- Within probability theory, there is a growing interest in computation with imprecise probabilities.
- Computation with imprecise probabilities is an integral part of GTU. Following are a few simple examples.
- Can these examples be dealt with through the use of existing theories of imprecise probabilities—theories which are based on bivalent-logic-based probability theory? My contention is that the answer is: No

SIMPLE EXAMPLES

- X is a real value random variable. Usually, X is much larger than approximately a, and much smaller than approximately b, where a and b are real numbers, with a < b. What is the expected value of X?

- f is a function from reals to reals described as: If X is small then Y is small; if X is medium then Y is large; if X is large then Y is small. What is the maximum of f?

CONTINUED

- A small glass jar contains balls of various sizes. Most are small and a few are large. What is the probability that a ball drawn at random is neither small nor large?

- Overeating causes obesity.

  Overeating and obesity cause high blood pressure.

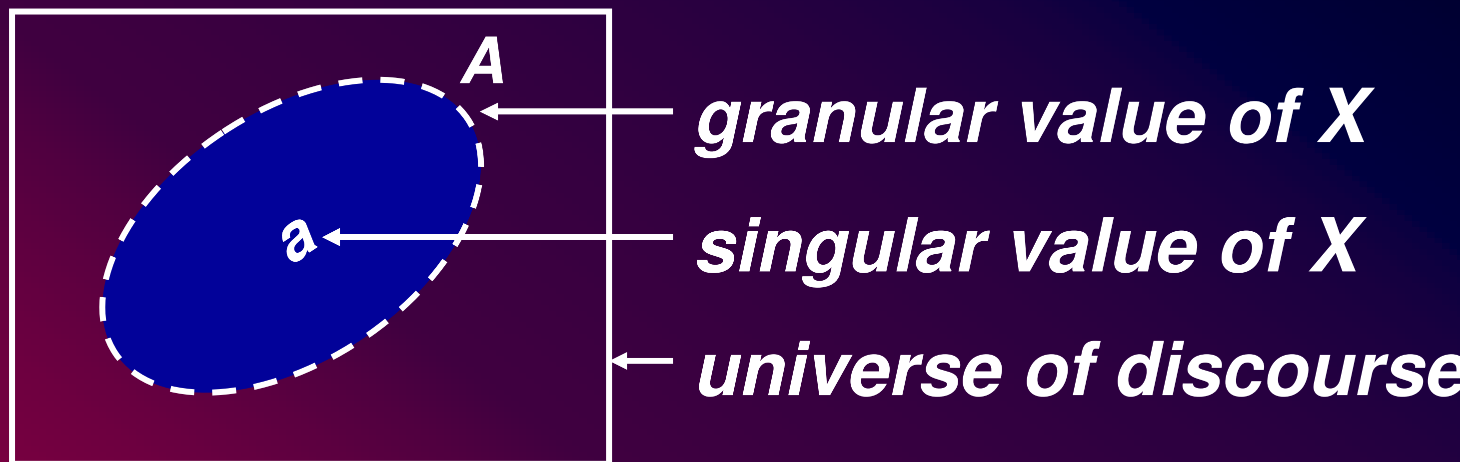
  I overeat. What is the probability that I will develop high blood pressure?

VALUATION

- *Assignment of a value to a variable*

singular	X	$\longleftarrow$	*number*
granular	X	$\longleftarrow$	*interval*
	X	$\longleftarrow$	*probability distribution*
	X	$\longleftarrow$	*possibility distribution*
	X	$\longleftarrow$	*word (linguistic variable)*
	X	$\longleftarrow$	*proposition (Computing with Words)*
	X	$\longleftarrow$	*system of propositions (NL-Computation)*

SINGULAR AND GRANULAR VALUES



	singular	granular
unemployment	7.3%	high
temperature	102.5	very high
blood pressure	160/80	high

- a granular value is defined by a generalized constraint

KEY POINT

- A basic difference between GTU and bivalent-logic-based theories of uncertainty relates to the role of natural languages. In GTU, semantics of natural languages plays a pivotal role.

- To compute with information described in natural languages, GTU employs the machinery of NL-Computation. The centerpiece of NL-Computation is the concept of a generalized constraint.

THE NEED FOR PRECISIATION OF NATURAL LANGUAGE

- Natural languages are intrinsically imprecise. A prerequisite to computation with information described in natural language is precisiation of meaning. A prerequisite to precisiation of meaning is understanding.

Example:

p: Use with adequate ventilation

I understand what you mean but can you be more precise?

- Precisiation of meaning is the first step in NL-Computation

THE CONCEPTS OF PRECISIATION AND COINTENSIVE PRECISIATION

PRECISION IN VALUE AND PRECISION IN MEANING

- ***The concept of precision has a position of centrality in scientific theories. And yet, there are some important aspects of this concept which have not been adequately treated in the literature. One such aspect relates to the distinction between precision in value (v-precision) and precision in meaning (m-precision).***
- ***The same distinction applies to imprecision, precisiation and imprecisiation.***

CONTINUED

PRECISE

v-precise — **m-precise**

precise value — *precise meaning*

- *p: X is in the interval [a, b]. a and b are precisely defined real numbers*
- *p is v-imprecise and m-precise*

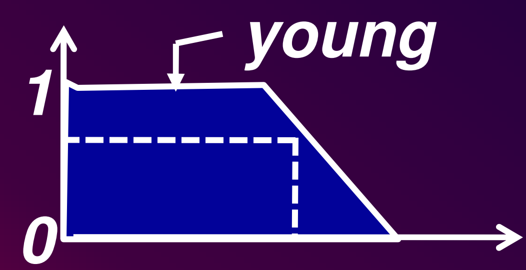
- *p: X is a Gaussian random variable with mean m and variance σ^2. m and σ^2 are precisely defined real numbers*
- *p is v-imprecise and m-precise*

PRECISIATION AND IMPRECISIATION

- ***A proposition, predicate, query or command may be precisiated or imprecisiated***
- ***Definition is a form of m-precisiation***

Example

young → **m-precisiation** →



Lily is 25 → **v-imprecisiation / m-imprecisiation** → *Lily is young*

Lily is young → **v-precisiation / m-precisiation** → *Lily is 25*

MODALITIES OF m-PRECISIATION

m-precisiation

mh-precisiation

mm-precisiation

human-oriented

machine-oriented (mathematically well-defined)

Example: bear market

mh-precisiation: declining stock market with expectation of further decline

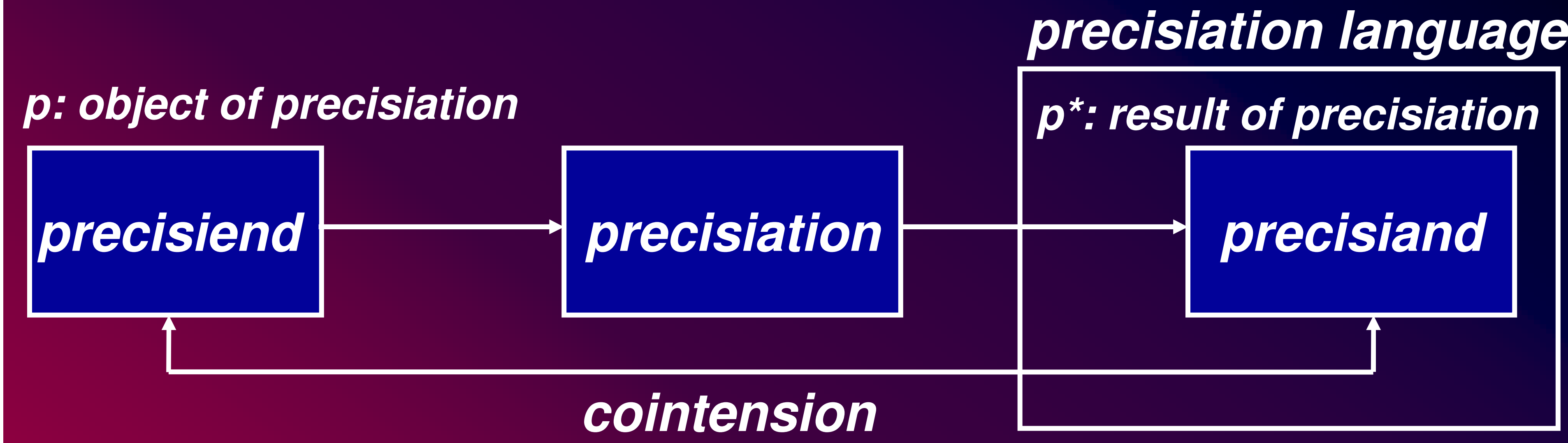
mm-precisiation: 30 percent decline after 50 days, or a 13 percent decline after 145 days. (Robert Shuster)

CONTINUED

- Risk $\xrightarrow{\text{mh-precisiation}}$ exposure to the chance of injury or loss

- Risk $\xrightarrow{\text{mm-precisiation}}$ expected value of loss function

BASIC CONCEPTS



precisiand = model of meaning
cointension = goodness of model of meaning
A precisiend has many precisiands. Pres(p) denotes the set of precisiands of p

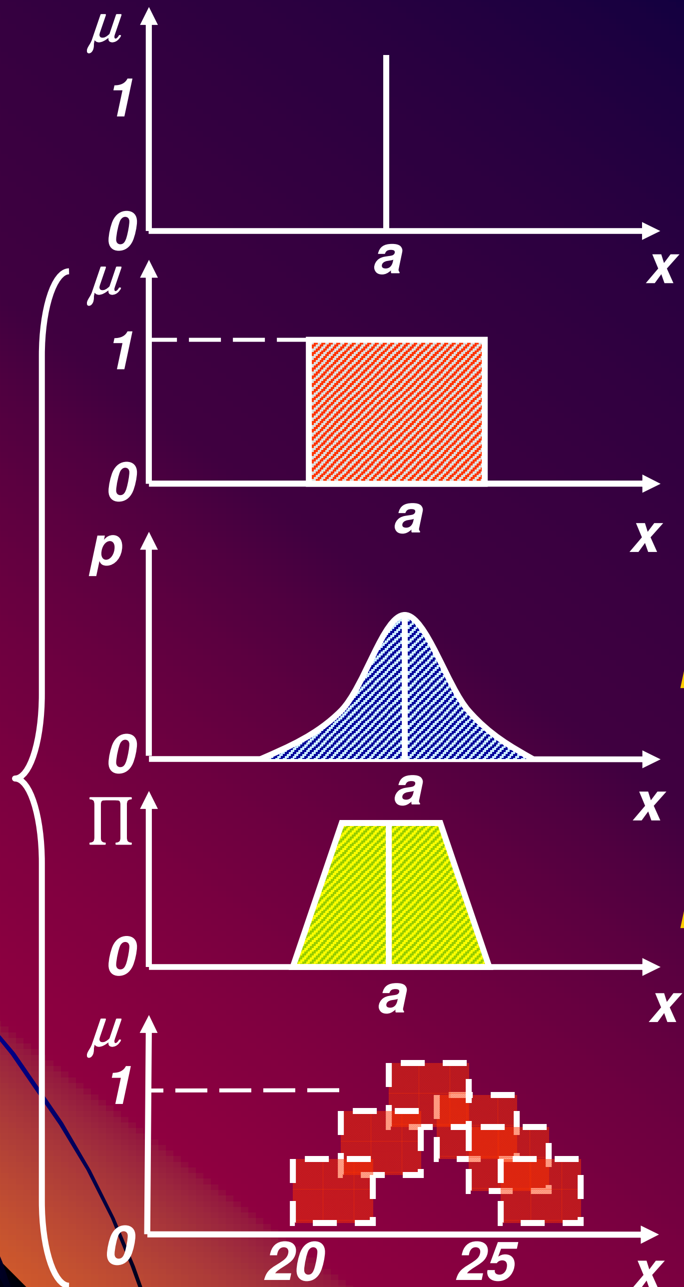
precisiation = translation into precisiation language

PRECISIATION OF "approximately a," *a

s-precisiation

cg-precisiation

g-precisiation



singleton

interval

probability distribution

possibility distribution

fuzzy graph

v-IMPRECISIATION

v-imprecisiation

- Imperative (forced)
- Intentional (deliberate)

imperative: value is not known precisely

intentional: value need not be known precisely

- *v-imprecisiation principle: Precision carries a cost. If there is a tolerance for imprecision, exploit it by employing v-imprecisiation to achieve lower cost, robustness, tractability, decision-relevance and higher level of confidence. Employ mm-precisiation to achieve computability.*
- *data compression and summarization are instances of v-imprecisiation*

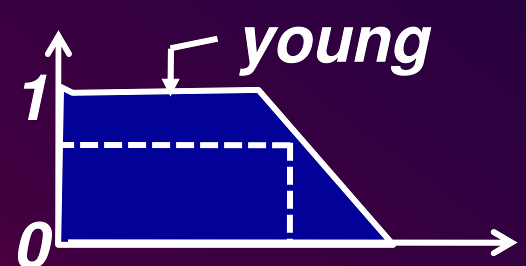
THE FUZZY LOGIC GAMBIT

p → ***v-imprecisiation*** → ***mm-precisiation*** →

reduction in cost (into v-imprecisiation)

achievement of computability (into mm-precisiation)

Lily is 25 → ***Lily is young*** →



Fuzzy logic gambit = v-imprecisiation followed by mm-precisiation

SCIENTIFIC PROGRESS

traditional

perceptions → words → numbers → computation/deduction

+

countertraditional

numbers → words → precisiated words → computation/deduction

Fuzzy Logic Gambit

CONTINUED

We achieve mm-precision by translation of natural language into a precisiation language. The fuzzy logic gambit opens the door to computation with information described in natural language and is the key idea in NL-Computation. More generally, in NL-Computation, the objects of computation are not values of variables but information about the values of variables, with the understanding that information is described in natural language. NL-Computation plays a pivotal role in the nontraditional view of fuzzy logic.

THE CONCEPT OF COINTENSION

- Informally, cointension is a matter of the goodness of model of meaning
- p, q are predicates or propositions
- CI(p,q): cointension of p and q: degree of match between the i-meanings of p and q
- q is cointensive w/n to p if GI(p, q) is high
- A definition is cointensive if CI(definiendum, definiens) is high
- The o-meaning of the definiendum is perception-based

THE CONCEPT OF COINTENSIVE PRECISIATION

- ***m-precisiation of a concept or proposition, p, is cointensive if p* is cointensive with p.***

Example: bear market

We classify a bear market as a 30 percent decline after 50 days, or a 13 percent decline after 145 days. (Robert Shuster)

This definition is clearly not cointensive

mm-PRECISIATION

- mm-precisiation is a prerequisite to computation/deduction with information described in natural language.

Basic questions

a) Given a proposition, p, how can p be cointesively mm-precisiated?

b) How can mm-precisiand of p be treated as an object of computation/deduction?

- In NL-Computation these questions are addressed through the use of generalized-constraint-based semantics.
- The centerpiece of generalized-constraint-based semantics is the concept of a generalized constraint.

THE CONCEPT OF A GENERALIZED CONSTRAINT AND ITS ROLE IN SEMANTICS OF NATURAL LANGUAGES

PREAMBLE

- *The concept of a generalized constraint is the centerpiece of generalized-constraint-based semantics. An outline of this concept if presented in the following.*

- *In scientific theories, representation of constraints is generally oversimplified. Oversimplification of constraints is a necessity because existing constrained definition languages have a very limited expressive power.*

CONTINUED

- ***The concept of a generalized constraint is intended to provide a basis for construction of a maximally expressive constraint definition language which can also serve as a meaning representation/precisiation language for natural languages.***

GENERALIZED CONSTRAINT (Zadeh 1986)

- ***Bivalent constraint (hard, inelastic, categorical:)***

  X ε ***C*** — ***constraining bivalent relation***

- ***Generalized constraint on X: GC(X)***

  $\boxed{GC(X):\ X\ isr\ R}$

  X — ***constrained variable***

  r — ***index of modality (defines semantics)***

  R — ***constraining non-bivalent (fuzzy) relation***

  $\underbrace{r: \varepsilon \mid = \mid \leq \mid \geq \mid \subset \mid \ldots}_{\text{bivalent}} \mid blank \mid p \mid v \mid u \mid rs \mid fg \mid ps \mid \ldots$

  primary

- ***open GC(X): X is free (GC(X) is a predicate)***
- ***closed GC(X): X is instantiated (GC(X) is a proposition)***

GENERALIZED CONSTRAINT—MODALITY r

X isr R

r: =	equality constraint: X=R is abbreviation of X is=R
r: ≤	inequality constraint: X ≤ R
r:⊂	subsethood constraint: X ⊂ R
r: blank	possibilistic constraint; X is R; R is the possibility distribution of X
r: v	veristic constraint; X isv R; R is the verity distribution of X
r: p	probabilistic constraint; X isp R; R is the probability distribution of X

Standard constraints: bivalent possibilistic, bivalent veristic and probabilistic

CONTINUED

r: bm bimodal constraint; X is a random variable; R is a bimodal distribution

r: rs random set constraint; X isrs R; R is the set-valued probability distribution of X

r: fg fuzzy graph constraint; X isfg R; X is a function and R is its fuzzy graph

r: u usuality constraint; X isu R means usually (X is R)

r: g group constraint; X isg R means that R constrains the attribute-values of the group

PRIMARY GENERALIZED CONSTRAINTS

- ***Possibilistic: X is R***
- ***Probabilistic: X isp R***
- ***Veristic: X isv R***

- ***Primary constraints are formalizations of three basic perceptions: (a) perception of possibility; (b) perception of likelihood; and (c) perception of truth***

- ***In this perspective, probability may be viewed as an attribute of perception of likelihood***

EXAMPLES: POSSIBILISTIC

- Monika is young ⟶ Age (Monika) is young

  X: Age (Monika); R: young

- Monika is much younger than Maria ⟶

  (Age (Monika), Age (Maria)) is much younger

  X: (Age (Monika), Age (Maria)); R: much younger

- most Swedes are tall

  ⟶ Count (tall.Swedes/Swedes) is most

  X: Count (tall.Swedes/Swedes); R: most

EXAMPLES: PROBABILISITIC

- X is a normally distributed random variable with mean m and variance σ^2 $\longrightarrow$

  X isp N(m, σ^2)

- X is a random variable taking the values u_1, u_2, u_3 with probabilities p_1, p_2 and p_3, respectively $\longrightarrow$

  X isp ($p_1 \backslash u_1 + p_2 \backslash u_2 + p_3 \backslash u_3$)

EXAMPLES: VERISTIC

- ***Robert is half German, quarter French and quarter Italian***

  Ethnicity (Robert) isv (0.5|German + 0.25|French + 0.25|Italian)

- ***Robert resided in London from 1985 to 1990***

  Reside (Robert, London) isv [1985, 1990]

STANDARD CONSTRAINTS

- ***Bivalent possibilistic: X ε C (crisp set)***
- ***Bivalent veristic: Ver(p) is true or false***
- ***Probabilistic: X isp R***
- ***Standard constraints are instances of generalized constraints which underlie methods based on bivalent logic and probability theory***

GENERALIZED CONSTRAINT LANGUAGE (GCL)

- GCL is generated by combination, qualification, propagation and counterpropagation of generalized constraints
- examples of elements of GCL
  - X/Age(Monika) is R/young (annotated element)
  - (X isp R) and (X,Y) is S)
  - (X isr R) is unlikely) and (X iss S) is likely
  - If X is A then Y is B
- the language of fuzzy if-then rules is a sublanguage of GCL

CLARIFICATION
LANGUAGE VS. LANGUAGE SYSTEM

- Language= (description system)
- Description system= (syntax, semantics)
- Language system= (description system, computation/deduction system)

Examples: Java is a language; Prologue is a language system; probability theory is al language system; fuzzy logic is a system of language systems.

- Generalized Constraint Language (GCL) is a language system.
- The rules of deduction in GCL are the rules which govern propagation and counterpropagation of generalized constraints.

EXTENSION PRINCIPLE

- The principal rule of deduction in NL-Computation is the Extension Principle (Zadeh 1965, 1975).

$$\frac{f(X) \text{ is } A}{g(X) \text{ is } B}$$

$$\mu_B(v) = \sup_u \mu_A(f(u))$$

subject to

$$v = g(u)$$

PROTOFORMAL DEDUCTION RULE

- *Syllogism*

$$\frac{\begin{array}{l} Q_1\ A\text{'s are } B\text{'s} \\ Q_2\ (A\&B)\text{'s are } C\text{'s} \end{array}}{Q_1 Q_2 A\text{'s are } (B\&C)\text{'s}}$$

Example

- *Overeating causes obesity* $\xrightarrow{\text{precisiation}}$ *most of those who overeat become obese*
- *Overeating and obesity cause high blood pressure* $\xrightarrow{\text{precisiation}}$ *most of those who overeat and are obese have high blood pressure*
- *I overeat and am obese. The probability that I will develop high blood pressure is most*^{2}

PROTOFORMAL DEDUCTION RULE

$$\frac{1/n\Sigma Count(G[H \text{ is } R]) \text{ is } Q}{1/n\Sigma Count(G[H \text{ is } S]) \text{ is } T}$$

$$\frac{\Sigma_i \mu_R(h_i) \text{ is } Q}{\Sigma_i \mu_S(h_i) \text{ is } T}$$

$$\mu_T(v) = sup_{h_1, \ldots, h_n}(\mu_Q(\Sigma_i \mu_R(h_i))$$

subject to

$$v = \Sigma_i \mu_S(h_i)$$

values of H: $h_1, \ldots, h_n$

GENERALIZED-CONSTRAINT-BASED SEMANTICS

- Generalized-constraint-based semantics suggests a novel, powerful approach to semantics of natural languages. It is generalized-constraint-based semantics that opens the door to computation with information described in natural language.
- The point of departure in generalized-constraint-based semantics is the fundamental thesis of fuzzy logic.

Information = generalized constraint

Basically, what this means is that information about a variable, X, may be viewed as a constraint on the values which X can take.

CONTINUED

- A proposition, p, is a carrier of information.
- A consequence of the fundamental thesis is the meaning postulate.

  meaning of p = generalized constraint
- In NL-Computation, the meaning of p is equated with its mm-precisiand. More specifically

proposition → $\boxed{\text{mm-precisiation}}$ → X is R (closed)

instantiated

predicate → $\boxed{\text{mm-precisiation}}$ → X is R (open)

free

MEANING POSTULATE—A RATIONALE

- *A proposition, p, may be viewed as an answer to a question, q.*
- *A question can be expressed as: What is the value of X? Where X is explicit or implicit in p.*
- *A generalized constraint may be interpreted as an answer to a question. From this it follows that the answer to q may be expressed as a generalized constraint.*

X isr R

- *In general X and R are implicit in p. In this sense, the meaning of p may be expressed as a generalized constraint in which X and R are defined procedurely.*
- *Note that X is a variable that is focused on but is not uniquely determined by X. For this reason, X is referred to as a focal variable.*

PRECISIATION AND DEDUCTION

- What should be stressed is that mm-precisiation is not the final goal. It is a preliminary to computation/deduction. The roles of mm-precisiation and computation/deduction are illustrated in the following.

PRECISIATION AND COMPUTATION/DEDUCTION—EXAMPLE

- *p: most Swedes are tall*

  p: ΣCount(tall.Swedes/Swedes) is most*
- *q: How many are short?*

further precisiation

X(h): height density function (not known)

X(h)du: fraction of Swedes whose height is in [h, h+du], $a \le h \le b$

$$\int_a^b X(h)du = 1$$

CONTINUED

- fraction of tall Swedes: $\int_a^b X(h)\mu_{tall}(h)dh$
- constraint on X(h)

$\int_a^b X(h)\mu_{tall}(h)dh$ is most

granular value

$$\pi(X) = \mu_{most}\left(\int_a^b X(h)\mu_{tall}(h)dh\right)$$

CONTINUED

deduction:

$$\frac{\int_a^b X(h)\mu_{tall}(h)dh \text{ is most} \longleftarrow \text{given}}{\int_a^b X(h)\mu_{short}(h)dh \text{ is ? } Q \longleftarrow \text{needed}}$$

solution:

$$\mu_Q(v) = \sup_X(\mu_{most}(\int_a^b X(h)\mu_{tall}(h)dh))$$

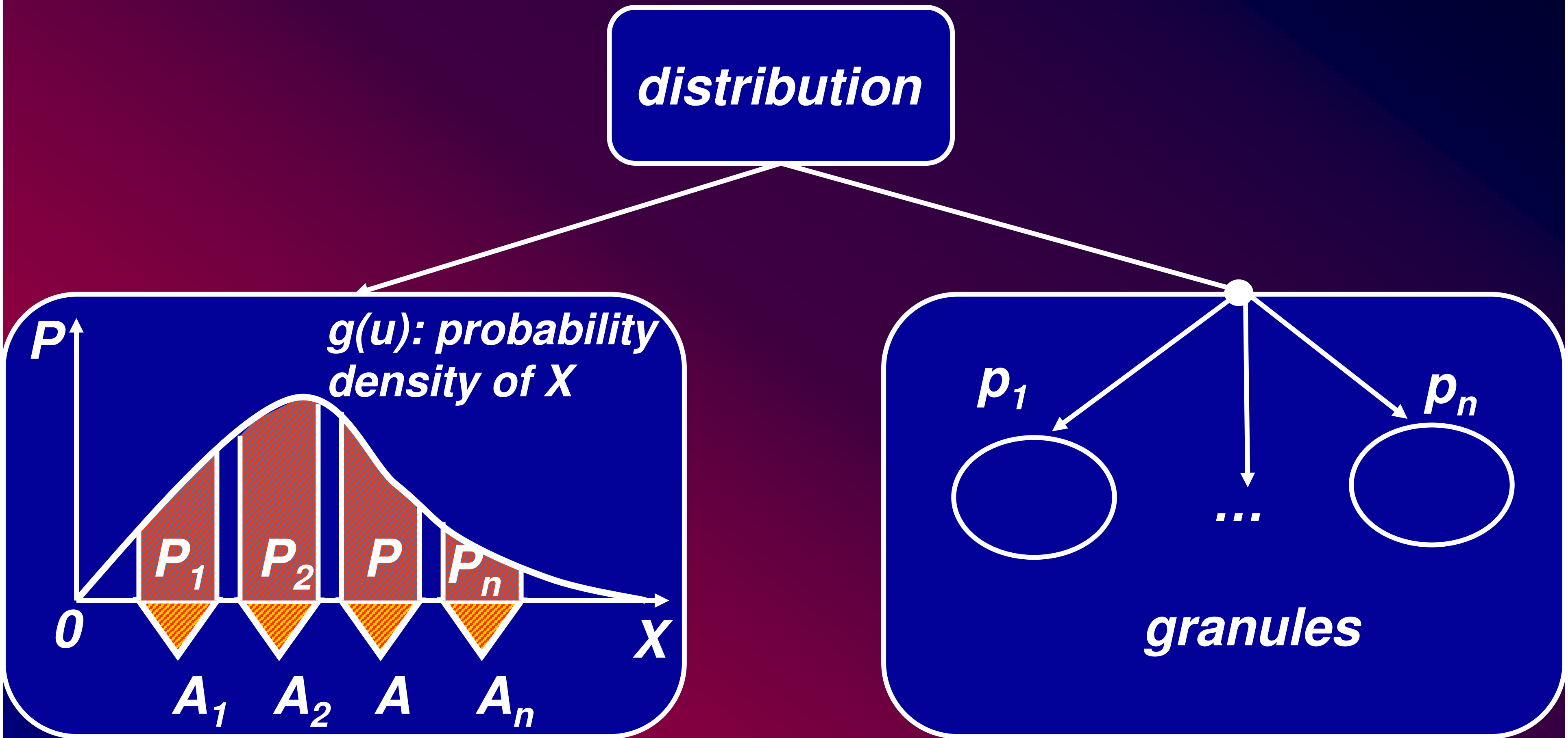
subject to

$$v = \int_a^b X(h)\mu_{short}(h)dh$$

$$\int_a^b X(h)dh = 1$$

PROBABILITY MODULE

GRANULAR VS. GRANULE-VALUED DISTRIBUTIONS

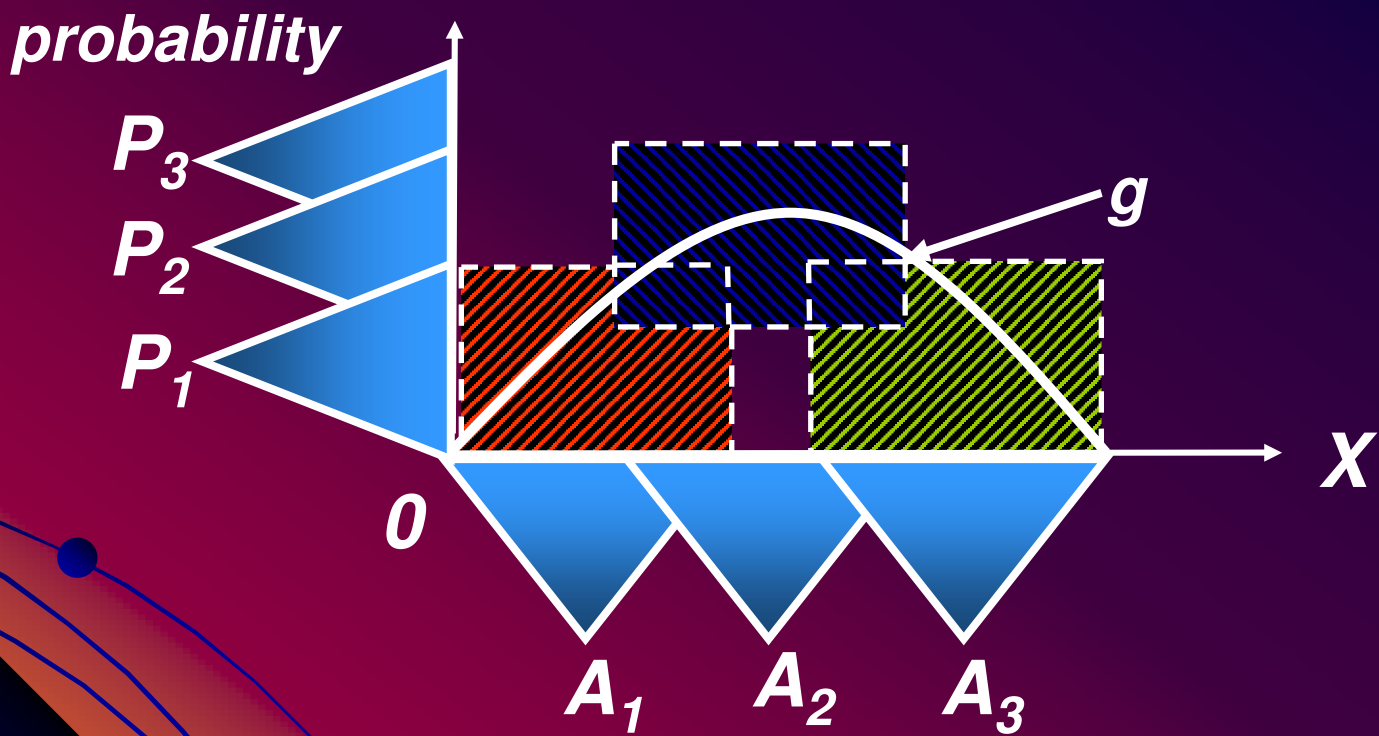


possibility distribution of probability distributions

probability distribution of possibility distributions

GRANULAR DISTRIBUTION (PERCEPTION-BASED PROBABILITY DISTRIBUTION)

X is a real-valued random variable

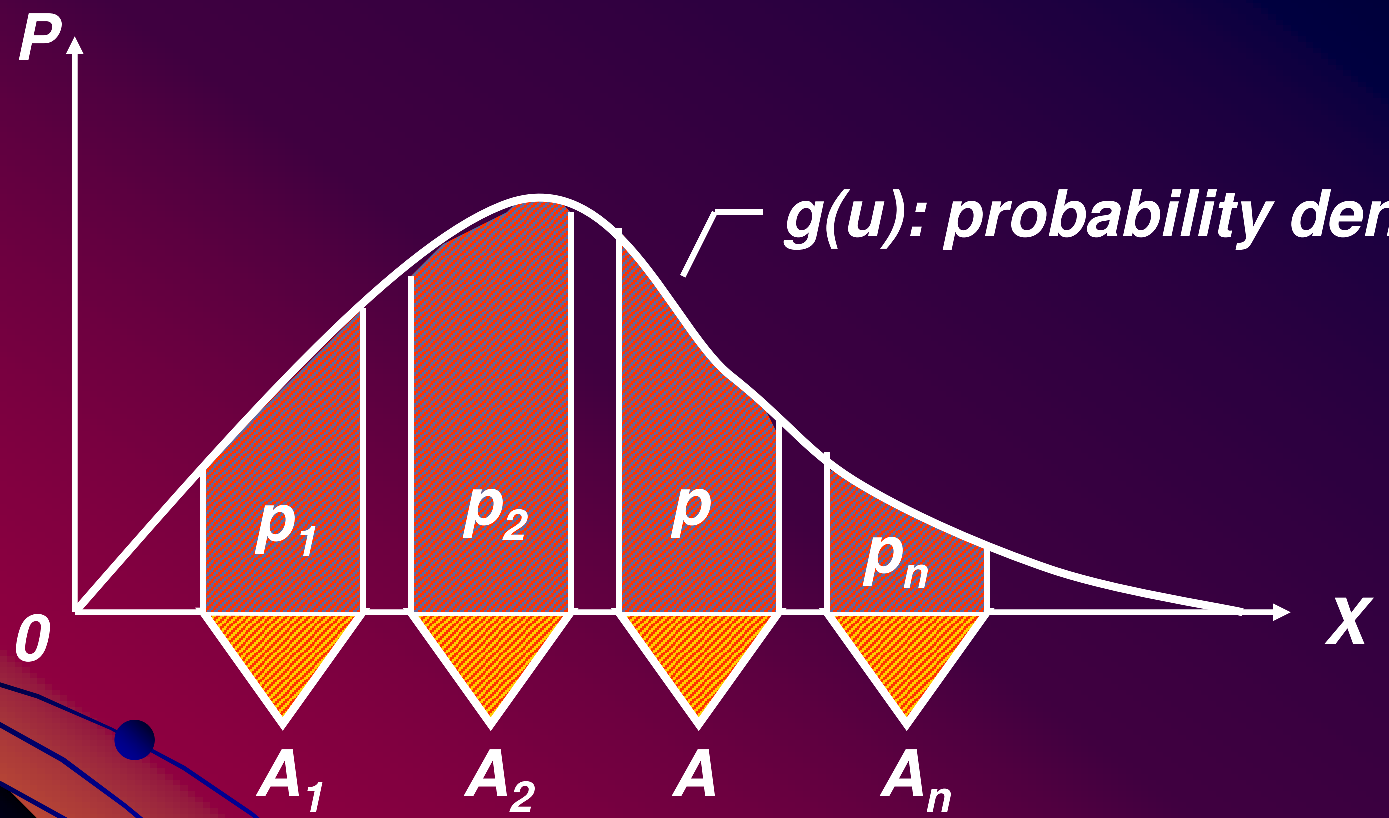


BMD: $P(X) = P_{i(1)} \backslash A_1 + P_{i(2)} \backslash A_2 + P_{i(3)} \backslash A_3$

Prob $\{X \text{ is } A_i\}$ is $P_{j(i)}$

P(X)= low\small + high\medium + low\large

INTERPOLATION OF GRANULAR DISTRIBUTION



p_i is P_i : granular value of p_i , i=1, …, n

(P_i , A_i) , i=1, …, n are given

A is given

(?P, A)

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INTERPOLATION MODULE AND PROBABILITY MODULE

$$\frac{\text{Prob }\{X \text{ is } A_i\} \text{ is } P_i \quad , i = 1, \ldots, n}{\text{Prob }\{X \text{ is } A\} \text{ is } Q}$$

$$\mu_Q(v) = sup_g(\mu_{P_1}(\int_U \mu_{A_1}(u)g(u)du) \wedge \cdots \wedge$$

$$\mu_{P_n} \int_U \mu_{P_n}(\int_U \mu_{A_n}(u)g(u)du))$$

subject to

$$U = \int_U \mu_A(u)g(u)du$$

EXAMPLE

- *Probably it will take about two hours to get from San Francisco to Monterey, and it will probably take about five hours to get from Monterey to Los Angeles. What is the probability of getting to Los Angeles in less than about seven hours?*

*BMD: (probably, *2) + (probably, *5)*

$$\underset{X}{\uparrow} \qquad \underset{Y}{\uparrow}$$

$$Z = X + Y$$

$$\underset{w}{\uparrow} \quad \underset{u}{\uparrow} \quad \underset{v}{\uparrow}$$

$$p_Z(w) = \int p_X(u) p_Y(w-u) du$$

CONTINUED

query: $\int p_Z(w)\mu_{\leq\circ *7}(w)dw$ is ?A

qri:

$$\Pi_{p_X} = \mu_{probably}(\int \mu_{*2}(u)p_X(u)du)$$

$$\Pi_{p_Y} = \mu_{probably}(\int \mu_{*5}(v)p_Y(v)dv)$$

$$\mu_A(t) = sup_{p_X, p_Y}(\Pi_X \wedge \Pi_Y)$$

subject to:

$$t = \int p_X(w)\mu_{\leq\circ *7}(w)dw$$

SUMMATION

- ***A basic difference between GTU and bivalent-logic-based theories of uncertainty relates to the role of natural languages. In GTU, semantics of natural languages plays a pivotal role. The underlying reason is that GTU's capability to compute with information described in natural language is a key feature—a feature which enables GTU to deal with perception-based information about probabilities, events and relations.***
- ***Another basic difference relates to the conceptual framework of GTU. In GTU, the basic concepts, e.g., the concepts of independence are defined, for the most part, through the use of PNL (Precisiated Natural Language). As a consequence, most of the basic concepts in GTU are context-dependent. All existing theories of uncertainty may be viewed as specializations of GTU.***

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